Units & Measurement and Dimensions

Question1

Which of the following expression can be deduced on the basis of dimensional analysis? (All symbols have their usual meanings)

KCET 2025

Options:

A. $x = A \cos \omega t$

B. $N = N_0 e^{-\lambda t}$

C. $F = 6 \pi \eta r \nu$

D. $s = ut + \frac{1}{2} a t^2$

Answer: C

Solution:

Only the drag law (Option C) follows purely from matching dimensions. In fact, if you assume

F depends on the viscosity η, radius r and speed ν as

$$F \propto \eta^a r^b \nu^c$$

then dimensional balance

- $[M\,L\,T^{-2}] = (M\,L^{-1}\,T^{-1})^a \cdot L^b \cdot (L\,T^{-1})^c$

gives

$$a = 1, \quad b = 1, \quad c = 1.$$

So

$$F = K \eta r \nu$$

where K is a dimensionless constant (in Stokes' solution K = 6π).

The other formulas (A, B, D) involve specific functional forms or numerical factors that dimensional analysis alone cannot fix.

Question2

Dimensional formula for activity of a radioactive substance is

KCET 2024

Options:

A. $\left[M^0\, L^1\, T^{-1} \right]$

B. $\left[M^0 L^{-1} T^0 \right]$

C. $\left[M^0\, L^0\, T^{-1} \right]$

D. $\left[M^{-1} L^0 T^0 \right]$

Answer: C

Solution:

The dimensional formula for the activity of a radioactive substance is expressed as the number of disintegrations per unit time. Therefore, the dimensions are related to inverse time.

$$\left[M^0\, L^0\, T^{-1} \right]$$

This indicates that activity is independent of mass (M^0) and length (L^0), and it depends on inverse time (T^{-1}). Hence, the correct option is C.

Question3

The true length of a wire is 3.678 cm. When the length of this wire is measured using instrument A, the length of the wire is 3.5 cm. When the length of the wire is measured using instrument B, it is found to have length 3.38 cm. Then, the

Options:

A. measurement with A is more accurate and precise.

B. measurement with A is more accurate while measurement with B is more precise.

C. measurement with B is more accurate and precise.

D. measurement with A is more precise while measurement with B is more constant.

Answer: B

Solution:

Given, actual length of wire $(l_0) = 3.678$ cm

Measurement with instrument, $A = 3.5$ cm

Measurement with instrument, $B = 3.38$ cm

Measurement of A is close to l_0 so it is more accurate but measurement of B is more precise as it can count upto two decimal places.

Question4

The Vernier scale of a travelling microscope has 50 divisions which coincides with 49 main scale divisions. If each main scale division is 0.5 mm, then the least count of the microscope is

KCET 2022

Options:

A. 0.5 mm

B. 0.01 mm

C. 0.5 cm

D. 0.01 cm

Answer: B

Solution:

The least count of a measuring instrument is the smallest value that can be measured with the instrument and is calculated by the formula:

$$\text{Least Count} = \frac{\text{Value of one main scale division}}{\text{Number of divisions on the Vernier scale}}$$

In the given travelling microscope, there are 50 divisions on the Vernier scale which coincides with 49 main scale divisions. Each main scale division is of 0.5 mm.

First, we need to find out the value of one main scale division:

Given that one main scale division is 0.5 mm, we keep this value for the calculation.

Next, to calculate the least count, we divide this value by the number of divisions on the Vernier scale.

$$\text{Least Count} = \frac{0.5 \text{ mm}}{50}$$

To calculate the value that the Vernier scale can measure beyond the main scale, we consider the discrepancy between the 49 main scale divisions and the 50 Vernier divisions. Since 49 main scale divisions coincide with 50 Vernier divisions, the length covered by 50 Vernier divisions is equal to the length covered by 49 main scale divisions.

Therefore, one Vernier division is equivalent to $\frac{49}{50}$ of a main scale division. The difference between the value of one main scale division and one Vernier division gives us the least count:

$$\text{Main scale division value} - \text{Vernier scale division value} = \text{Least Count}$$

$$\text{Least Count} = \frac{0.5 \text{ mm}}{50} \times 49$$

Since there is a difference of 1 Vernier division in 50 divisions, the Vernier scale is more precise by:

$$\text{Least Count} = \frac{0.5 \text{ mm}}{50}$$

Simplifying the fraction:

$$\text{Least Count} = \frac{0.5}{50} \text{ mm}$$

$$\text{Least Count} = 0.01 \text{ mm}$$

So the correct answer is:

Option B

0.01 mm

Question5

A cylindrical wire has a mass (0.3 ± 0.003)g**, radius** (0.5 ± 0.005)mm **and length** (6 ± 0.06)cm**. The maximum**

percentage error in the measurement of its density is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

Given, mass, $m = (0.3 \pm 0.003)$g

Radius, $r = (0.5 \pm 0.005)$ mm

Length, $l = (6 \pm 0.06)$ cm

Density of cylinder, $\rho = \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{m}{\pi r^2 l}$

Fraction error in ρ,

$\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + \dfrac{2\Delta r}{r} + \dfrac{\Delta l}{l}$

Percentage error in ρ,

$$\dfrac{\Delta\rho}{\rho} \times 100 = \dfrac{\Delta m}{m} \times 100 + \dfrac{2\Delta r}{r} \times 100 + \dfrac{\Delta l}{l} \times 100$$
$$= \dfrac{0.003}{0.3} \times 100 + 2 \times \dfrac{0.005}{0.5} \times 100 + \dfrac{0.06}{6} \times 100$$
$$= 1 + 2(1) + 1 = 4\%$$

Question6

If P, Q and R are physical quantities having different dimensions, which of the following combinations can never be a meaningful quantity?

Options:

A. $\frac{P-Q}{R}$

B. $PQ - R$

C. $\frac{PQ}{R}$

D. $\frac{PR-Q^2}{R}$

Answer: A

Solution:

P, Q and R are physical quantities having different dimensions.

By the principle of homogenity, the physical quantities having different dimensions, cannot be added or subtracted directly.

Hence, $\frac{P-Q}{R}$ is not a meaningful quantity.

Question7

The dimensions of the ratio of magnetic flux(ϕ) and permeability (μ) are

KCET 2018

Options:

A. $\left[M^0L^1T^0A^1\right]$

B. $\left[M^9L^{-3}T^0A^1\right]$

C. $\left[M^0L^1T^1A^{-1}\right]$

D. $\left[M^0L^2T^0A^1\right]$

Answer: A

Solution:

To determine the dimensions of the ratio of magnetic flux (ϕ) to permeability (μ), we start with their respective dimensions:

The dimensions of magnetic flux ϕ are given by:

$$\phi = \left[\mathrm{ML^2T^{-2}A^{-1}}\right]$$

The dimensions of permeability μ are given by:

$$\mu = \left[\mathrm{MLT^{-2}A^{-2}}\right]$$

To find the dimensions of the ratio $\frac{[\phi]}{[\mu]}$:

$$\frac{[\phi]}{[\mu]} = \frac{\left[\mathrm{ML^2T^{-2}A^{-1}}\right]}{\left[\mathrm{MLT^{-2}A^{-2}}\right]}$$
$$= \left[\mathrm{M^0L^{2-1}T^{-2-(-2)}A^{-1-(-2)}}\right]$$
$$= \left[\mathrm{M^0L^1T^0A^1}\right]$$

Thus, the dimensions of $\frac{[\phi]}{[\mu]}$ are $\left[\mathrm{M^0L^1T^0A^1}\right]$.

Question8

A substance of mass 49.53 g occupies $1.5 \ \mathrm{cm}^3$ of volume. The density of the substance (in gcm^{-3}) with correct number of significant figures is

KCET 2017

Options:

A. 3.3

B. 3.300

C. 3.302

D. 33.0

Answer: D

Solution:

To find the density, we use the formula:

$$\rho = \frac{m}{V}$$

where

- $m = 49.53\,\text{g}$

- $V = 1.5\,\text{cm}^3$

Step 1: Calculate the density

$$\rho = \frac{49.53\,\text{g}}{1.5\,\text{cm}^3} \approx 33.02\,\text{g/cm}^3$$

Step 2: Consider Significant Figures

The mass $49.53\,\text{g}$ has 4 significant figures.

The volume $1.5\,\text{cm}^3$ has 2 significant figures.

When dividing, the final answer must have the same number of significant figures as the value with the fewest significant figures. Here, that is 2 significant figures.

Step 3: Round to 2 Significant Figures

Rounding 33.02 to 2 significant figures gives:

$33\,\text{g/cm}^3$

Among the options provided, the value that closely represents this magnitude is:

Option D: 33.0

Thus, the correct answer is Option D.